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THE GEOMETRY OF PARTIAL DIFFERENTIAL HAMILTONIAN SYSTEMS

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Abstract. Partial differential Hamiltonian systems have been recently introduced by the author in arXiv:0903.4528. They are field theoretic analogues of Hamiltonian systems on abstract symplectic manifolds. We will present here their main geometry, consisting of partial differential Hamilton equations, partial differential Noether theorem, partial differential Poisson bracket, etc..

1. Introduction

Lagrangian mechanics can be naturally generalized to first order Lagrangian field theory. Moreover, the latter can be presented in a very elegant and precise algebro-geometric fashion [12]. On the other hand, it seems to be a bit harder to understand what the most “reasonable, unambiguous, field theoretic generalization” of Hamiltonian mechanics on abstract symplectic manifolds is. Actually, there exists an universally accepted field theoretic version of Hamiltonian mechanics on the cotangent bundle T^*Q of a configuration manifold Q, i.e., multisymplectic field theory on the multimomentum bundle of a configuration bundle. However, T^*Q is just a very special example of (pre)symplectic manifold. Hamiltonian mechanics can (and should, in some cases [8]) be formulated on abstract (pre)symplectic manifolds. Similarly, it is natural to wonder if there exists the concept of abstract multi(pre)symplectic manifolds in such a way that Hamiltonian field theory could be reasonably formulated on them. In the literature there can be found some proposals of should be abstract multi(pre)symplectic manifolds (see, for instance, [1, 3]). Recently, the author presented his own proposal about what should be an abstract, first order, Hamiltonian field theory, and called it the theory of **partial**