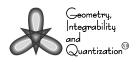
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## A FORMULAE FOR THE SPECTRAL PROJECTIONS OF TIME OPERATOR

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**Abstract.** In this paper, we study the one-level Friedrichs model by using the quantum time super-operator that predicts the excited state decay inside the continuum. Its survival probability decays exponentially in time.

## 1. Introduction

In this paper we shall study the concept of survival probability of an unstable quantum system introduced in [6] and we shall test it in the Friedrichs model [7]. The survival probability should be a monotonically decreasing time function and this property could not exists in the framework of the usual Weisskopf-Wigner approach [1, 2, 8, 11]. It could only be properly treated if it is defined through an observable T (time operator) whose eigenprojections provide the probability distribution of the time of decay. The equation defining the **time operator** T is

$$U_{-t}TU_t = T + tI \tag{1}$$

where  $U_t$  is the unitary group of states evolution. It is known that such an operator cannot exist when the evolution is governed by the Schrödinger equation, since the Hamiltonian has a bounded spectrum from below, and this contradicts the equation

$$[H,T] = iI \tag{2}$$

in the Hilbert space of pure states  $\mathcal{H}$ . However, the time operator T can exist under some conditions, for mixed states. They can be embedded [3, 6, 12] in the "Liouville space", denoted  $\mathcal{L}$ , that is the space of Hilbert-Schmidt operators  $\rho$  on  $\mathcal{H}$  such that  $\operatorname{Tr}(\rho^* \rho) < \infty$ , equipped with the scalar product  $<\rho, \rho' >= \operatorname{Tr}(\rho^* \rho')$ .