RICHARDSON-GAUDIN ALGEBRAS AND THE EXACT SOLUTIONS OF THE PROTON-NEUTRON PAIRING

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Abstract. Many exactly solvable models are based on Lie algebras. The pairing interaction is important in nuclear physics and its exact solution for identical particles in non-degenerate single-particle levels was first given by Richardson in 1963. His solution and its generalization to Richardson-Gaudin quasi-exactly solvable models have attracted the attention of many contemporary researchers and resulted in the exact solution of the isovector pn-pairing within the $SO(5)$ RG-model and the equal strength spin-isospin pn-pairing within the $SO(8)$ RG-model. Basic properties of the RG-models are summarized and possible applications to nuclear physics are emphasized.

1. Introduction

Symmetry is one of the most important paradigms in modern physics. Any Lie group and its Lie algebra have a naturally defined action on a product of spaces (representations). Thus, they are very suitable for a multi-particle system with an underling symmetry. Usually, this means that the relevant operators are well defined for one particle as well as for any number of particles. This allows one to find exact solutions to a problem, with a given underlining symmetry, by referring to the relevant representation theory of the symmetry group at place. As a result many exactly solvable models are build using Lie algebra representation theory. A well known examples are the theories with $SO(3)$ and $SU(2)$ rotational symmetry, the Elliott’s $U(3)$ symmetry model [4–7], the Wigner’s $SU(4)$ spin-isospin symmetry [17], and many more that play a major role in nuclear physics. For example, the $SO(8)$ and $Sp(6)$ Ginnocchio models, the Fermion Dynamical Symmetry Models (FDSM), and the three dynamical symmetries of the Interacting Boson Model (IBM).