Twelfth International Conference on
Geometry, Integrability and Quantization
June 4–9, 2010, Varna, Bulgaria
Ivaïlo M. Mladenov, Gaetano Vilasi
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2011, pp 237–243

NEW PARAMETERIZATIONS OF THE CASSINIAN OVALS

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Abstract. Here we present a general scheme which leads effectively to the reconstruction of any plane curve whose curvature is specified by a function of the radial coordinate. As a concrete example we have derived two new parametrizations of the Cassinian ovals.

1. Introduction

Surprising or not it turns out that the curvature of a lot of the famous plane curves such as conic sections, Bernoulli's lemniscate [8, 15], Cassinian ovals [2, 12, 16], Delaunay surfaces [13, 17] and their generalization [7], Euler's elastica [5, 11], Sturmian spirals [4, 17], and many others, depends solely on the distance from a certain point or a line in the Euclidean plane. Let us remind also that the most fundamental existence and uniqueness theorem in the theory of plane curves states that a curve is uniquely determined (up to Euclidean motion) by its curvature given as a function of its arc-length (see [3, p. 296] or [19, p. 37]). The simplicity of the situation however is quite elusive because in many cases it is impossible to find the sought-after curve explicitly. Having this in mind, it is clear that if the curvature is given by a function of its position the situation is even more complicated. Viewing the Frenet-Serret equations as a ficticious dynamical system in [22] it was proven that when the curvature is given just as a function of the distance from the origin the problem can always be reduced to quadratures. The cited result should not be considered as entirely new because Singer [21] has already shown that in some cases it is possible that such curvature gets an interpretation of a central potential in the plane and therefore the trajectories could be found by the standard procedures in