NEW PARAMETERIZATIONS OF THE CASSINIAN OVALS

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Abstract. Here we present a general scheme which leads effectively to the
reconstruction of any plane curve whose curvature is specified by a function
of the radial coordinate. As a concrete example we have derived two new
parametrizations of the Cassinian ovals.

1. Introduction

Surprising or not it turns out that the curvature of a lot of the famous plane curves
such as conic sections, Bernoulli’s lemniscate [8, 15], Cassinian ovals [2, 12, 16],
Delaunay surfaces [13, 17] and their generalization [7], Euler’s elastica [5, 11],
Sturmian spirals [4, 17], and many others, depends solely on the distance from a
certain point or a line in the Euclidean plane. Let us remind also that the most
fundamental existence and uniqueness theorem in the theory of plane curves states
that a curve is uniquely determined (up to Euclidean motion) by its curvature given
as a function of its arc-length (see [3, p. 296] or [19, p. 37]). The simplicity of the
situation however is quite elusive because in many cases it is impossible to find the
sought-after curve explicitly. Having this in mind, it is clear that if the curvature is
given by a function of its position the situation is even more complicated. Viewing
the Frenet-Serret equations as a fictitious dynamical system in [22] it was proven
that when the curvature is given just as a function of the distance from the origin the
problem can always be reduced to quadratures. The cited result should not be con-
considered as entirely new because Singer [21] has already shown that in some cases
it is possible that such curvature gets an interpretation of a central potential in the
plane and therefore the trajectories could be found by the standard procedures in