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MOTION OF CHARGED PARTICLES IN TWO-STEP NILPOTENT LIE GROUPS*

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Abstract. We formulate the equation of motion of a charged particle in a Riemannian manifold with a closed two form. Since a two-step nilpotent Lie group has natural left-invariant closed two forms, it is natural to consider the motion of a charged particle in a simply connected two-step nilpotent Lie groups with a left invariant metric. We study the behavior of the motion of a charged particle in the above spaces.

1. Introduction

Let Ω be a closed two-form on a connected Riemannian manifold $(M, \langle\ ,\ \rangle)$, where $\langle\, ,\rangle$ is a Riemannian metric on M. We denote by $\bigwedge^m(M)$ the space of m-forms on M. We denote by $\iota(X) : \bigwedge^m(M) \to \bigwedge^{m-1}(M)$ the interior product operator induced from a vector field X on M, and by $\mathcal{L} : T(M) \to T^*(M)$, the Legendre transformation from the tangent bundle $T(M)$ over M onto the cotangent bundle $T^*(M)$ over M, which is defined by

$$\mathcal{L} : T(M) \to T^*(M), \quad u \mapsto \mathcal{L}(u), \quad \mathcal{L}(u)(v) = \langle u, v\rangle\ , \quad u, v \in T(M). \tag{1}$$

A curve $x(t)$ in M is referred as a *motion of a charged particle under electromagnetic field* Ω, if it satisfies the following second order differential equation

$$\nabla_{\dot{x}}\dot{x} = -\mathcal{L}^{-1}(\iota(\dot{x})\Omega) \tag{2}$$

where ∇ is the Levi-Civita connection of M. Here $\nabla_{\dot{x}}\dot{x}$ means the acceleration of the charged particle. Since $-\mathcal{L}^{-1}(\iota(\dot{x})\Omega)$ is perpendicular to the direction $\dot{x}$ of the movement, $-\mathcal{L}^{-1}(\iota(\dot{x})\Omega)$ means the **Lorentz force**. The speed $|\dot{x}|$ is a conservative constant for a charged particle. When $\Omega = 0$, then the motion of a

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