MOTION OF CHARGED PARTICLES IN TWO-STEP NILPOTENT LIE GROUPS*

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Abstract. We formulate the equation of motion of a charged particle in a Riemannian manifold with a closed two form. Since a two-step nilpotent Lie group has natural left-invariant closed two forms, it is natural to consider the motion of a charged particle in a simply connected two-step nilpotent Lie groups with a left invariant metric. We study the behavior of the motion of a charged particle in the above spaces.

1. Introduction

Let Ω be a closed two-form on a connected Riemannian manifold (M, ⟨ , ⟩), where ⟨ , ⟩ is a Riemannian metric on M. We denote by Λ^m(M) the space of m-forms on M. We denote by τ(X) : Λ^m(M) → Λ^{m-1}(M) the interior product operator induced from a vector field X on M, and by L : T(M) → T^*(M), the Legendre transformation from the tangent bundle T(M) over M onto the cotangent bundle T^*(M) over M, which is defined by

L : T(M) → T^*(M), u → L(u), L(u)(v) = ⟨u, v⟩, u, v ∈ T(M). (1)

A curve x(t) in M is referred as a motion of a charged particle under electromagnetic field Ω, if it satisfies the following second order differential equation

\nabla_\dot{x} \ddot{x} = -L^{-1}(\dot{x})Ω \tag{2}

where \nabla is the Levi-Civita connection of M. Here \nabla_\dot{x} \ddot{x} means the acceleration of the charged particle. Since \(-L^{-1}(\dot{x})Ω\) is perpendicular to the direction \dot{x} of the movement, \(-L^{-1}(\dot{x})Ω\) means the Lorentz force. The speed \|\dot{x}\| is a conservative constant for a charged particle. When Ω = 0, then the motion of a charged particle follows the geodesics of M.