DIFFERENTIAL GEOMETRY OF MOVING SURFACES AND ITS RELATION TO SOLITONS

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Abstract. In this article we present an introduction in the geometrical theory of motion of curves and surfaces in $\mathbb{R}^3$, and its relations with the nonlinear integrable systems. The working frame is the Cartan’s theory of moving frames together with Cartan connection. The formalism for the motion of curves is constructed in the Serret-Frenet frames as elements of the bundle of adapted frames. The motion of surfaces is investigated in the Gauss-Weingarten frame. We present the relations between types of motions and nonlinear equations and their soliton solutions.

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1. Introduction

Realistic models for many-body or collective interactions involve nonlinear dynamics therefore a large part of interesting and intriguing phenomena cannot be