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EINSTEIN METRICS WITH TWO-DIMENSIONAL KILLING LEAVES AND THEIR APPLICATIONS IN PHYSICS

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Abstract. Solutions of vacuum Einstein's field equations, for the class of pseudo-Riemannian four-metrics admitting a non Abelian two dimensional Lie algebra of Killing fields, are explicitly described. When the distribution orthogonal to the orbits is completely integrable and the metric is not degenerate along the orbits, these solutions are parameterized either by solutions of a transcendental equation (the tortoise equation), or by solutions of a linear second order differential equation in two independent variables. Metrics, corresponding to solutions of the tortoise equation, are characterized as those that admit a three dimensional Lie algebra of Killing fields with two dimensional leaves. Metrics, corresponding to the case in which the commutator of the two Killing fields is isotropic, represent nonlinear gravitational waves.

1. Introduction

The aim of this paper is to illustrate some interesting and, in some sense, surprising physical properties of special solutions of Einstein field equations belonging to the larger class of Einstein metrics invariant for a non-Abelian Lie algebra of Killing vector fields generating a two dimensional distribution.

Some decades ago, by using a suitable generalization of the *Inverse Scattering Transform*, Belinsky and Sakharov [3] were able to determine four-dimensional Ricci-flat Lorentzian metrics invariant for an Abelian two dimensional Lie algebra of Killing vector fields such that the distribution $\mathcal{D}^{\perp}$ orthogonal to the one, say $\mathcal{D}$, generated by the Killing fields is transversal to $\mathcal{D}$ and Frobenius-integrable.

Thus, as a first step, it has been natural to consider [16] the problem of characterizing all gravitational fields g admitting a Lie algebra $\mathcal{G}$ of Killing fields such that