

A NATURAL GEOMETRIC FRAMEWORK FOR THE SPACE OF INITIAL DATA OF NONLINEAR PDES

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Abstract. The modern geometrical approach to nonlinear PDEs is the outcome of a nontrivial synthesis of differential calculus over commutative algebras and cohomological algebra in the context of infinite jet spaces. In this paper we propose a very natural generalization of the notion of a jet space, which allows to treat the space of initial data of a nonlinear PDE on the same footing as the space of its solutions.

1. Introduction

In spite of its age, Henneaux & Teitelboim’s book “Quantization of Gauge System” still stands as a sort of Bible for modern theoretical physicists. In the middle of it, the reader meets three simple prescriptions (ten lines overall) which allow to recast in a field-theoretical context the results of BRST formalism obtained so far. But soon comes a warning: “Although useful, this approach to the field theoretical case remains, however, rather formal as long as one does not specify precisely the functional space to which the relevant functionals should belong. This turns out, in general, to be a complicated task” ([2], Chapter 12). The fact that deep field-theoretical results keep coming, regardless of the lack of solid mathematical foundations, might be taken as an evidence that any effort to find such foundations is, in fact, superfluous.

What theoretical physicists seem to ignore is that a robust geometrical language for nonlinear field theories *does exist*, but its comprehension is – to use the same words as the authors above – “a complicated task”. We will call such a language *Secondary Calculus*, following Vinogradov (see [10, 11] and references therein), but it should be stressed that, in spite of its remarkable achievements in the covariant description of nonlinear Lagrangian theories and PDEs, it is still a young