



HARMONIC SPHERES AND YANG–MILLS FIELDS

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Abstract. We study a relation between harmonic spheres in loop spaces of compact Lie groups and Yang–Mills fields on the Euclidean four-space \mathbb{R}^4 .

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