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QUADRATIC HAMILTON-POISSON SYSTEMS ON $\mathfrak{so}^*_-(3)$: CLASSIFICATION AND INTEGRATION

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Abstract. We classify, under affine equivalence, the quadratic Hamilton-Poisson systems on the Lie-Poisson space $\mathfrak{so}^*_-(3)$. For the simplest strictly inhomogeneous quadratic system, we find explicit expressions for the integral curves in terms of Jacobi elliptic functions.

1. Introduction

Quadratic Hamilton-Poisson systems on Lie-Poisson spaces have received attention from several authors in recent years (see, e.g., [3, 6–8, 14]). Equivalence of quadratic Hamilton-Poisson systems has been considered by Tudoran [12,13]. The use of equivalence (in reducing to normal forms) has proved promising for the analysis of various classes of such systems (see, e.g., [3, 9]).

Homogeneous quadratic Hamilton-Poisson systems on the orthogonal Lie-Poisson space $\mathfrak{so}^*_-(3)$ have been treated in [5,9] were both stability and integration were addressed. In this paper we classify, under affine equivalence, the homogeneous and *inhomogeneous* quadratic Hamilton-Poisson systems on $\mathfrak{so}^*_-(3)$ and an exhaustive list of normal forms are exhibited. (Two systems are said to be affinely equivalent if their associated vector fields are compatible with an affine isomorphism.) Among the inhomogeneous systems obtained as normal forms, we integrate the simplest one. Three qualitatively different cases are identified for this system. In each case explicit expressions for the integral curves are found in terms of Jacobi elliptic functions.