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A RECURSION OPERATOR FOR THE GEODESIC FLOW ON N-DIMENSIONAL SPHERE

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Abstract. For a completely integrable system, the way of finding first integrals is not formulated in general. A new characterization for integrable systems using the particular tensor field is investigated which is called a recursion operator. A recursion operator T for a vector field Δ is a diagonizable (1, 1)-type tensor field, invariant under Δ and has vanishing Nijenhuis torsion. One of the important property of T is that T gives constants of the motion (the sequence of first integrals) for the vector field Δ. The purpose of this paper is to discuss a recursion operator T for the geodesic flow on S^n.

1. Introduction

For a completely integrable system, the way of finding the first integrals is not formulated in general.

Liouville proved that a system with n degrees of freedom is integrable by quadratures when there exist n independent first integrals in involution (cf. [1]).

In classical mechanics, *a completely integrable system* in the sense of Liouville are called simply *an integrable system*.

Integrable systems related to the recursion operator were characterized in many papers $[2, 3, 6, 8]$ written since 1980.

There the integrable system is characterized by the recursion operator T in the Hamiltonian dynamical system on the cotangent bundle $T^*\mathcal{M}$ of a manifold $\mathcal{M}$.

The recurtion operator T is a diagonalizable $(1, 1)$-tensor field which satisfies certain conditions. In particular, it can be written in the following form if we choose