SYMMETRIES OF HAMILTONIAN DYNAMICAL SYSTEMS,
MOMENTUM MAPS AND REDUCTIONS

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Abstract. This text presents some basic notions in symplectic geometry,
Poisson geometry, Hamiltonian systems, Lie algebras and Lie groups actions
on symplectic or Poisson manifolds, momentum maps and their use for the
reduction of Hamiltonian systems. It should be accessible to readers with a
general knowledge of basic notions in differential geometry. Full proofs of
many results are provided.

CONTENTS

1. Introduction .............................................................. 12
2. Symplectic Manifolds .................................................. 13
  2.1. Definition and Elementary Properties .......................... 13
  2.2. Examples of Symplectic Manifolds ............................. 14
  2.3. Remarkable Submanifolds of a Symplectic Manifold .......... 16
  2.4. Hamiltonian Vector Fields on a Symplectic Manifold ........ 17
  2.5. The Poisson bracket ................................................. 21
3. Poisson Manifolds ...................................................... 23
  3.1. The Inception of Poisson Manifolds ............................. 23
  3.2. Definition and Structure of Poisson Manifolds ............... 24
  3.3. Some Properties of Poisson Manifolds ........................ 27
  3.4. Examples of Poisson Manifolds ................................ 31
4. Symplectic, Poisson and Hamiltonian Actions .................... 32
  4.1. Actions on a Smooth Manifold .................................. 32
  4.2. Poisson, Symplectic and Hamiltonian Actions ............... 35
  4.3. Some Properties of Momentum Maps ........................... 37
  4.4. Actions of a Lie Group on its Cotangent Bundle ............ 43
5. Reduction of Hamiltonian Systems with Symmetries .............. 45
  5.1. The Marsden-Weinstein Reduction Procedure .................. 46