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DIFFUSION UNDER GEOMETRICAL CONSTRAINT

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Abstract. Here we discus the diffusion of particles in a curved tube. This kind of transport phenomenon is observed in biological cells and porous media. To solve such a problem, we discuss the three dimensional diffusion equation with a confining wall forming a thinner tube. We find that the curvature appears in a effective diffusion coefficient for such a quasi-one-dimensional system. As an application to higher dimensional case, we discuss the diffusion in a curved surface with thickness. In this case the diffusion coefficient changes to the tensor form depending on the mean and Gaussian curvatures. Then the diffusion flow can be interpreted as usual flow plus anomalous flow. The anomalous flow shows not only the diffusion but also the concentration depending on mean and Gaussian curvatures, and also it includes the flow proportional to the gradient of Gaussian curvature.

1. Introduction

The particle motion on a given curved manifold (surface) is old but interesting problem. It shows not only the check of general relativity, but also the physical phenomena in a scale around us. But in latter case, precisely saying, it is the curved line with girth ($M_1 \times \mathbb{R}^2$) and the curved surface with thickness ($M_2 \times \mathbb{R}$) embedded in the three-dimensional Euclidean space $\mathbb{R}^3$. Usually we neglect the effect of the girth and the thickness, and we identify the system as the motion on a curved line and curved surface. This is true when the length parameters $\{l_1, l_2, \cdots l_N\}$ in M and thickness ϵ of the system satisfies the conditions

$$l_i >> \epsilon, \qquad i = 1, 2, \dots N \tag{1}$$

where l_i are the parameters with dimension of the length in M. One such example is $1/\kappa$, where κ is the local curvature of M. But when the thickness (or grith) ϵ is comparable to the inverse of curvature, it is no longer true even though all of other length parameters are satisfying the condition (1). In such a case, we can construct