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A CONSTRUCTION OF A RECURSION OPERATOR FOR SOME SOLUTIONS OF EINSTEIN FIELD EQUATIONS

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Abstract. The $(1,1)$-tensor field on symplectic manifold that satisfies some integrability conditions is called a recursion operator. It is known the recursion operator is a characterization for integrable systems, and gives constants of motion for integrable systems. We construct recursion operators for the geodesic flows of some solutions of Einstein equation like Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman metrics.

1. Introduction

Liouville proved that when a Hamiltonian system with n degrees of freedom on a $2n$-dimensional phase space has n independent first integrals in involution the system is integrable by quadratures (cf [1]).

On the other hand, de Filippo, Marmo, Salerno and Vilasi (see e.g. [2, 3, 6, 10] and [11]) proposed a new characterization of integrable systems. Let us consider a vector field on $\mathcal{M}^{2n}$.

Theorem 1 ([11]). *A vector field X is separable, integrable and Hamiltonian for certain symplectic structure when X admits an invariant, mixed, diagonalizable $(1,1)$-tensor field T with vanishing Nijenhuis torsion and doubly degenerate eigenvalues without stationary points. Then, the vector field X is a separable and completely integrable Hamiltonian system with respect to the symplectic structure in the sense of Liouville.*

Now, the operator T in Theorem 1 is called a **recursion operator**. Several examples of recursion operators e.g., the harmonic oscillator and the Kepler dynamics, are given in [6] and [11]. In this paper we consider geodesic flows for the