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LOCALITY OF THE CONSERVATION LAWS FOR THE SOLITON EQUATIONS RELATED TO CAUDREY-BEALS-COIFMAN SYSTEM VIA THE THEORY OF RECURSION OPERATORS

ALEXANDAR YANOVSKI

Department of Mathematics and Applied Mathematics, University of Cape Town 7700 Rondebosch, Cape Town, South Africa

Abstract. We consider the hierarchies of Nonlinear Evolution Equations related to auxiliary problem of Caudrey-Beals-Coifman type. We give a proof that the conservation laws for these equations have local densities based on the theory of the generating operators related to the Caudrey-Beals-Coifman linear problem.

1. Introduction

This article is about the theory of the so-called soliton equations (completely integrable equations). Their characteristic property is that they can be cast into the so called Lax form, or zero curvature form, that is, as compatibility condition (Lax representation) $[L, A] = 0$ for two linear systems

$$\begin{aligned} L\psi &= (\mathrm{i}\partial_x - U(q, q_x, \ldots, \lambda))\,\psi = 0 \\ A\psi &= (\mathrm{i}\partial_t - V(q, q_x, \ldots, \lambda))\,\psi = 0. \end{aligned} \tag{1}$$

Here U, V are matrix functions, depending on the spectral parameter λ and on a set of 'potential functions' $q(x,t) \equiv (q_1(x,t), q_2(x,t), \ldots, q_N(x,t))$ and their spatial derivatives $q_x, q_{xx}, \ldots$ and t is the time. The equation $[L, A] = 0$ is equivalent to an equation (system) of the type $q_t = f(q, q_x, \ldots)$ which is the soliton equation itself, [7, 12, 29]. Usually the first of the equations in (1), that is $L\psi = 0$, is fixed and called auxiliary linear problem. Changing the second one we obtain hierarchies of nonlinear evolution equations (NLEEs) related to the linear problem $L\psi = 0$. Each hierarchy is usually named by some of the equations belonging to it.