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PRE-SYMPLECTIC STRUCTURE ON THE SPACE OF CONNECTIONS

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Abstract. Let X be a four-manifold with boundary three-manifold M. We shall describe (i) a pre-symplectic structure on the sapce $\mathcal{A}(X)$ of connections on the bundle $X \times \mathrm{SU}(n)$ that comes from the canonical symplectic structure on the cotangent space $T^*\mathcal{A}(X)$. By the boundary restriction of this pre-symplectic structure we obtain a pre-symplectic structure on the space $\mathcal{A}_0^\flat(M)$ of flat connections on $M \times \mathrm{SU}(n)$ that have null charge.

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1. Introduction

Let X be an oriented Riemannian four-manifold with boundary $M = \partial X$. For the trivial principal bundle $P = X \times \mathrm{SU}(n)$ we denote by $\mathcal{A}(X)$ the space of irrreducible connections on X. The following theorems are proved.

Theorem 1. *Let $P = X \times \mathrm{SU}(n)$ be the trivial $\mathrm{SU}(n)-$principal bundle on a four-manifold X. There exists a canonical pre-symplectic structure on the space of irreducible connections $\mathcal{A}(X)$ given by the two-form*

$$\sigma_A^s(a,b) = \frac{1}{8\pi^3}\int_X \mathrm{Tr}[(ab-ba)F_A] - \frac{1}{24\pi^3}\int_M \mathrm{Tr}[(ab-ba)A]$$

for $a, b \in T_A\mathcal{A}(X)$.

Theorem 2. *Let ω be a two-form on $\mathcal{A}(M)$ defined by*

$$\omega_A(a,b) = -\frac{1}{24\pi^3}\int_M \mathrm{Tr}[(ab-ba)A]$$