PRE-SYMPLECTIC STRUCTURE ON THE SPACE OF CONNECTIONS

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Abstract. Let $X$ be a four-manifold with boundary three-manifold $M$. We shall describe (i) a pre-symplectic structure on the space $A(X)$ of connections on the bundle $X \times SU(n)$ that comes from the canonical symplectic structure on the cotangent space $T^*A(X)$. By the boundary restriction of this pre-symplectic structure we obtain a pre-symplectic structure on the space $A^\flat_0(M)$ of flat connections on $M \times SU(n)$ that have null charge.

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1. Introduction

Let $X$ be an oriented Riemannian four-manifold with boundary $M = \partial X$. For the trivial principal bundle $P = X \times SU(n)$ we denote by $A(X)$ the space of irreducible connections on $X$. The following theorems are proved.

**Theorem 1.** Let $P = X \times SU(n)$ be the trivial $SU(n)$-principal bundle on a four-manifold $X$. There exists a canonical pre-symplectic structure on the space of irreducible connections $A(X)$ given by the two-form

$$
\sigma^*_A(a, b) = \frac{1}{8\pi^3} \int_X \text{Tr}[(ab - ba)F_A] - \frac{1}{24\pi^3} \int_M \text{Tr}[(ab - ba)A]
$$

for $a, b \in T_AA(X)$.

**Theorem 2.** Let $\omega$ be a two-form on $A(M)$ defined by

$$
\omega_A(a, b) = -\frac{1}{24\pi^3} \int_M \text{Tr}[(ab - ba)A]
$$

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