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ON THE TRAJECTORIES OF $\mathrm{U}(1)$-KEPLER PROBLEMS

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Abstract. The classical $\mathrm{U}(1)$-Kepler problems at level $n \geq 2$ were formulated, and their trajectories are determined via an idea similar to the one used by Kustaanheimo and Stiefel in the study of Kepler problem. It is found that a non-colliding trajectory is an ellipse, a parabola or a branch of hyperbola according as the total energy is negative, zero or positive, and the complex orientation-preserving linear automorphism group of $\mathbb{C}^n$ acts transitively on both the set of elliptic trajectories and the set of parabolic trajectories.

MSC: 53D20, 53Z05, 70F05, 70G65, 70H06
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1. Introduction

The quantum $\mathrm{U}(1)$-Kepler problems, which are higher dimensional generalizations of the MICZ-Kepler problems [9, 16], have been introduced and studied [10] for quiet a while. Their intimate connection with representation theory [1], especially local theta-correspondence [3], has been demonstrated in [10] as well. However, their corresponding classical models, though not difficult to be formulated, seem to be difficult to solve, that is why there is a significant delay of the current work. The clue to solve these classical models finally came after a closer examination of [4, 7, 8] and [12–15].

To formulate these classical models, we start with the euclidean Jordan algebra $\mathrm{H}_n(\mathbb{C})$ of complex hermitian matrices of order n. (Euclidean Jordan algebras were initially introduced by Jordan [5], and were subsequently classified by Jordan, von Neuman and Wigner [6]. A good reference for euclidean Jordan algebras is [2].) Next, we introduce the space $\mathcal{C}_1$ of rank one semi-positive elements in $\mathrm{H}_n(\mathbb{C})$. Thirdly, we observe that there are two canonical structures on the space $\mathcal{C}_1$: