

VISIBLE ACTIONS ON GENERALIZED FLAG VARIETIES AND A GENERALIZATION OF THE CARTAN DECOMPOSITION

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Abstract. With the aim of uniform treatment of multiplicity-free representations of Lie groups, T. Kobayashi introduced the theory of visible actions on complex manifolds.

Our main results give a classification of triples (G, H, L) for a compact Lie group G and its Levi subgroups H, L , which satisfy $G = HBL$. Here, B is a subset of a Chevalley–Weyl involution σ -fixed points subgroup G^σ of G . The point here is that one decomposition $G = LBH$ produces three strongly visible actions on generalized flag varieties, and thus three finite-dimensional multiplicity-free representations (Kobayashi’s triunity principle).

Furthermore, we can also prove that the visibility of actions of compact Lie groups, the existence of a decomposition $G = LBH$ and the multiplicity-freeness property of finite-dimensional tensor product representations are all equivalent.

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1. Introduction

Let G be a connected compact Lie group and L, H Levi subgroups of G . Then, the homogeneous spaces G/H and G/L are generalized flag varieties. In this article, we give a classification of triples (G, L, H) such that the following three group-actions are strongly visible

$$L \curvearrowright G/H, \quad H \curvearrowright G/L, \quad \text{diag}(G) \curvearrowright (G \times G)/(H \times L). \quad (1)$$