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NEW ADVANCES IN THE STUDY OF GENERALIZED WILLMORE SURFACES AND FLOW

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Abstract. In this paper we study a Generalized Willmore flow for graphs and its numerical applications. First, we derive the time dependent equation which describes the geometric evolution of a Generalized Willmore flow in the graph case. This equation is recast in divergence form as a coupled system of second order nonlinear PDEs. Furthermore, we study finite element numerical solutions for steady-state cases obtained with the help of the FEMuS library (Finite Element Multiphysics Solver). We use automatic differentiation (AD) tools to compute the exact Jacobian of the coupled PDE system subject to Dirichlet boundary conditions.

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1. Introduction

Let M be a smooth immersed surface in $\mathbb{R}^3$. We consider the Generalized Willmore energy functional

$$W = \int_M (aH^2 + b)\,\mathrm{d}S$$

where $a = 2k_c$ represents the double of the usual bending rigidity and b is the surface tension coefficient. The term $\mathrm{d}S$ is the area element with respect to the induced metric. Then, the corresponding Euler-Lagrange equation is given by

$$\Delta H + 2(H^2 - K - \epsilon)H = 0$$