A HARMONIC ENDOMORPHISM IN A SEMI-RIEMANNIAN CONTEXT

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Abstract. On the total space of the cotangent bundle $T^*M$ of a Riemannian manifold $(M, h)$ we consider the natural Riemann extension $\tilde{g}$ with respect to the Levi-Civita connection of $h$. In this setting, we construct on $T^*M$ a new para-complex structure $P$, whose harmonicity with respect to $\tilde{g}$ is characterized here by using the reduction of $\tilde{g}$ to the (classical) Riemann extension.

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1. Introduction

Let $M$ be a connected smooth $n$-dimensional manifold and let $T^*M$ be its cotangent bundle. We suppose that the manifold $M$ is endowed with a symmetric linear connection $\nabla$. In [12], Patterson and Walker introduced the (classical) Riemann extension that was generalized by Sekizawa and Kowalski to natural Riemann extension, which is a semi-Riemannian metric of signature $(n, n)$, on the total space of $T^*M$, (see [14] and [11]). Later, Bejan and Kowalski [5] characterized harmonic functions with respect to the natural Riemann extension $\tilde{g}$ on $T^*M$. Also, the natural Riemann extension is a special class of modified Riemann extensions which is studied in [7] and [10].

Harmonicity is a very interesting topic in some mathematical fields, such as differential geometry, analysis, partial differential equations, theoretical physics and so on. We recall that a $C^2$- map $\varphi: (N, h) \rightarrow (\tilde{N}, \tilde{h})$ between (semi-)Riemannian