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A HARMONIC ENDOMORPHISM IN A SEMI-RIEMANNIAN CONTEXT

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Abstract. On the total space of the cotangent bundle T^*M of a Riemannian manifold (M, h) we consider the natural Riemann extension $\bar{g}$ with respect to the Levi-Civita connection of h. In this setting, we construct on T^*M a new para-complex structure P, whose harmonicity with respect to $\bar{g}$ is characterized here by using the reduction of $\bar{g}$ to the (classical) Riemann extension.

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1. Introduction

Let M be a connected smooth n-dimensional manifold and let T^*M be its cotangent bundle. We suppose that the manifold M is endowed with a symmetric linear connection ∇. In [12], Patterson and Walker introduced the (classical) Riemann extension that was generalized by Sekizawa and Kowalski to natural Riemann extension, which is a semi-Riemannian metric of signature (n, n), on the total space of T^*M, (see [14] and [11]). Later, Bejan and Kowalski [5] characterized harmonic functions with respect to the natural Riemann extension $\bar{g}$ on T^*M. Also, the natural Riemann extension is a special class of modified Riemann extensions which is studied in [7] and [10].

Harmonicity is a very interesting topic in some mathematical fields, such as differential geometry, analysis, partial differential equations, theoretical physics and so on. We recall that a C^2- map $\varphi : (N, h) \to (\bar{N}, \bar{h})$ between (semi-)Riemannian