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VECTOR-PARAMETER FORMS OF SU(1,1), SL(2, $\mathbb{R}$) AND THEIR CONNECTION TO SO(2,1)

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Abstract. The *Cayley* maps for the Lie algebras $\mathfrak{su}(1,1)$ and $\mathfrak{so}(2,1)$ converting them into the corresponding Lie groups $\mathrm{SU}(1,1)$ and $\mathrm{SO}(2,1)$ along their natural vector-parameterizations are examined. Using the isomorphism between $\mathrm{SU}(1,1)$ and SL(2, $\mathbb{R}$), the vector-parameterization of the latter is also established. The explicit form of the covering map $\mathrm{SU}(1,1) \to \mathrm{SO}(2,1)$ and its sections are presented. Using the so developed vector-parameter formalism, the composition law of $\mathrm{SO}(2,1)$ in vector-parameter form is extended so that it covers compositions of all kinds of elements including also those that can not be parameterized properly by regular $\mathrm{SO}(2,1)$ vector-parameters. The latter are characterized and it is shown that they can be represented by $\mathrm{SU}(1,1)$ vector parameters with pseudo length equal to minus four. In all cases of compositions inside $\mathrm{SO}(2,1)$, criteria for determination of their type (elliptic, parabolic, hyperbolic) have been presented. On the base of the vector-parameter formalism the problem of taking a square root in SO(2,1) is solved explicitly. Also, an analogue of Cartan's theorem about the decomposition of orthogonal matrix of order n into product of at most n reflections is formulated and proved for the subset of hyperbolic elements of the group of pseudo-orthogonal matrices from $\mathrm{SO}(2,1)$.