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BOUR SURFACE COMPANIONS IN SPACE FORMS

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Abstract. In this paper, we give explicit parametrizations for Bour type surfaces in various three-dimensional space forms, using Weierstrass-type representations. We also determine classes and degrees of some Bour type zero mean curvature surfaces in three-dimensional Minkowski space.

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1. Introduction

Minimal surfaces in three-dimensional Euclidean space $\mathbb{R}^3$ isometric to rotational surfaces were first introduced by Bour [2] in 1862. All such minimal surfaces are given via the well-known Weierstrass representation for minimal surfaces by choosing suitable data depending on a parameter m, as shown by Schwarz [15]. They are called Bour's minimal surfaces $\mathfrak{B}_m$ of value m. Furthermore, when m is an integer greater than one, $\mathfrak{B}_m$ become algebraic, that is, there is an implicit polynomial equation satisfied by the three coordinates of $\mathfrak{B}_m$, see also [5, 13, 18].

Kobayashi [9] gave an analogous Weierstrass-type representation for conformal spacelike surfaces with mean curvature identically zero, called maximal surfaces, in three-dimensional Minkowski space $\mathbb{R}^{2,1}$. We remark that Magid [12] gave a Weierstrass-type representation for timelike surfaces with mean curvature identically zero, called timelike minimal surfaces, in $\mathbb{R}^{2,1}$, see also [8].

On the other hand, Lawson [10] showed that there is an isometric correspondence between constant mean curvature (CMC for short) surfaces in Riemannian space