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FUNCTIONALS ON TOROIDAL SURFACES

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Abstract. We show that the torus in $\mathbb{R}^3$ is a critical point of a sequence of functionals $\mathcal{F}_n$ $(n = 1, 2, 3, \ldots)$ defined over compact surfaces (closed membranes) in $\mathbb{R}^3$. When the Lagrange function $\mathcal{E}$ is a polynomial of degree n of the mean curvature H of the torus, the radii (a, r) of the torus are constrained to satisfy $\frac{a^2}{r^2} = \frac{n^2-n}{n^2-n-1}$, $n \geq 2$. A simple generalization of torus in $\mathbb{R}^3$ is a tube of radius r along a curve α which we call it toroidal surface (TS). We show that toroidal surfaces with non-circular curve α do not provide minimal energy surfaces of the functionals $\mathcal{F}_n$ $(n = 2, 3)$ on closed surfaces. We discuss possible applications of the functionals discussed in this work on cell membranes.

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1. Introduction

In the history of differential geometry there are some special subclasses of surfaces in $\mathbb{R}^3$, such as surfaces of constant Gaussian curvature, surfaces of constant mean curvature, minimal surfaces and the Willmore surfaces. These surfaces arise in many different branches of sciences. In particular, in various parts of theoretical physics (string theory, general theory of relativity), cell-biology and differential geometry [2,4–8,11–15,19–25]. All these special surfaces constitute critical points of certain functionals. Euler-Lagrange equations of these functionals are very complicated and difficult. There are some techniques developed to solve them, such as using the deformation of the Lax equations of the integrable equations so that it is possible to construct surfaces in $\mathbb{R}^3$ [4, 6, 7, 18] solving the Euler-Lagrange equations.