

EXTENDED HARMONIC MAPPINGS AND EULER-LAGRANGE EQUATIONS

KEIICHI KIKUCHI

*Department of Mathematics, School of Science, Tokai University, Hiratsuka
Kanagawa, 259-1292, Japan*

Abstract. Via the Lagrangian formalism, an example of extended harmonic CMC immersion and conservation laws are obtained.

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1. Introduction

We consider the Lagrangian formalism, where Lagrangians have the potential functions. In the previous papers [1–3], the periodicity of some families of S^1 -equivariant CMC (constant mean curvature) surfaces in the Berger sphere or the hyperbolic three-space was proved by making use of the conservation laws, in particular, we find that the potential functions of Lagrangians which correspond to S^1 -equivariant CMC- H surfaces contain the constant mean curvature H itself (see §3 and also [4]). Throughout the paper, we consider some extended harmonic mappings via Euler-Lagrange equations (Propositions 1 and 5). The extended harmonic mapping can be considered as a natural extension of harmonic mapping, since the potential function of corresponding Lagrangian to harmonic mapping is vanishing. We give examples of extended harmonic mapping and extended harmonic CMC- H immersion (§3). By using the conservation laws (Theorems 2, 3 and §6) with respect to the Hamiltonians, we investigate a certain geometric relationship between an extended harmonic mapping and a smooth mapping with vanishing tension field (Theorems 4 and 6).