LECTURES ON GEOMETRIC QUANTIZATION

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Abstract. These lectures notes are meant as an introduction to geometric quantization. In Section 1, I begin with presentation of the historical background of quantum mechanics. I continue with discoveries in the theory of representations of Lie groups, which lead to emergence of geometric quantization as a part of pure mathematics. This presentation is very subjective, flavored by my own understanding of the role of geometric quantization in quantum mechanics and representation theory. Section 2 is devoted to a review of geometry of Hamiltonian systems. Geometric quantization is discussed in the next two sections: prequantization in Section 3 and polarization in Section 4. In particular, I discuss geometric quantization with respect to polarizations given by Kähler structure, cotangent bundle projection and completely integrable system. More advanced topics, like metaplectic structure, pairing of polarizations, and commutation of quantization and reduction, are not included.

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