

## LOCAL AND NON-LOCAL CONSERVATION LAWS FOR QUADRATIC CONSTRAINED LAGRANGIANS AND APPLICATIONS TO COSMOLOGY

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**Abstract.** We study the existence of conservation laws in constrained systems described by quadratic Lagrangians; the type of which is encountered in mini-superspace cosmology. As is well known, variational symmetries lead to conserved quantities that can be used in the classical and quantum integration of a system. Additionally - and due to the parametrization invariance of such Lagrangians - conditional symmetries defined on phase space can lead to non-local integrals of motion. The latter may be of importance in various cosmological configurations. As an example we present the case of scalar field cosmology with an arbitrary potential.

**MSC:** 70H45, 70H06, 70G65

**Keywords:** Conditional symmetries, constrained systems, mini-superspace cosmology

### 1. Introduction

Symmetries at both the classical and the quantum level are of utmost importance in many physical theories. The same is also true for cosmology, especially in the context of a mini-superspace approximation. That is, when there can be constructed an equivalent mechanical system which exhibits the same dynamical evolution as the gravitational one. There is a series of works that deal with symmetries of these systems and how they are used to derive solutions or constrain the theory under consideration [2, 12, 13, 15]. When a mini-superspace approximation is adopted a constrained (or singular) system is obtained, meaning that not all of the equations of motion are independent. Usually, in the literature, a particular gauge fixing condition is being applied so as to treat these Lagrangians as regular. However,