CAYLEY MAP AND HIGHER DIMENSIONAL REPRESENTATIONS OF ROTATIONS

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Abstract. The embeddings of the $\mathfrak{so}(3)$ Lie algebra and the Lie group $\text{SO}(3)$ in higher dimensions is an important construction from both mathematical and physical viewpoint. Here we present results based on a program package for building the generating matrices of the irreducible embeddings of the $\mathfrak{so}(3)$ Lie algebra within $\mathfrak{so}(n)$ in arbitrary dimension $n \geq 3, n \neq 4k+2, k \in \mathbb{N}$ relying on the algorithm developed recently by Campoamor-Strursberg [3]. For the remaining cases $n = 4k + 2$ embeddings of $\mathfrak{so}(3)$ into $\mathfrak{so}(n)$ are also constructed. Besides, we investigate the characteristic polynomials of these $\mathfrak{so}(n)$ elements. We show that the Cayley map applied to $C \in \mathfrak{so}(n)$ is well defined and generates a subset of $\text{SO}(n)$. Furthermore, we obtain explicit formulas for the images of the Cayley map. The so obtained $\text{SO}(n)$ matrices are expressed as polynomials of $C$ whose coefficients are rational functions of the norm of the vector-parameter $c$. The composition laws are extracted for the cases $n = 4, 6$ and for the first case it is shown that via the Cayley map the isomorphism $\text{SU}(2) \cong \text{im Cay}_{4} \cup \{-I_{4}\}$ holds. Also, for $n = 4$ explicit formulas for the angular velocity matrices are derived. Comparisons between the results obtained via the exponential map and the Cayley map are made as well. In contrast to the case of the Cayley map, the results for the exponential map include either irrational or transcendental functions of the module of the vector-parameter.

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