

ON AUTO AND HETERO BÄCKLUND TRANSFORMATIONS FOR THE HÉNON-HEILES SYSTEMS

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Abstract. We consider a canonical transformation of parabolic coordinates on the plain associated with integrable Hénon-Heiles systems and suppose that this transformation together with some additional relations may be considered as a counterpart of the auto and hetero Bäcklund transformations.

MSC: 58J72, 70H06

Keywords: Auto and hetero Bäcklund transformations, Hénon-Heiles systems

1. Introduction

According classical definition by Darboux [5], a Bäcklund transformation between two given PDEs

$$E_1(u, x, t) = 0 \quad \text{and} \quad E_2(v, y, \tau) = 0$$

is a pair of relations

$$F_{1,2}(u, x, t, v, y, \tau) = 0 \tag{1}$$

and some additional relations between (x, t) and (y, τ) , which allow to get both equations $E_{1,2}$. The BT is called an auto-BT or a hetero-BT depending whether the two PDEs are the same or not. The hetero-BTs describe a correspondence between equations rather than a one-to-one mapping between their solutions [1, 12]. In the modern theory of partial differential equation Bäcklund transformations are seen also as a powerful tool in the discretization of PDEs [2].