QUANTIZATION OF LOCALLY SYMMETRIC KÄHLER MANIFOLDS

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Abstract. We introduce noncommutative deformations of locally symmetric Kähler manifolds. A Kähler manifold M is said to be a locally symmetric Kähler manifold if the covariant derivative of the curvature tensor is vanishing. An algebraic derivation process to construct a locally symmetric Kähler manifold is given. As examples, star products for noncommutative Riemann surfaces and noncommutative CP^N are constructed.

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1. Review of the Deformation Quantization with Separation of Variables

In this section, we review the deformation quantization with separation of variables to construct noncommutative Kähler manifolds.

An N-dimensional Kähler manifold M is described by using a Kähler potential. Let Φ be a Kähler potential and ω be a Kähler two-form

\[ \omega := i g_{kl} d z^k \wedge d \bar{z}^l, \quad g_{kl} := \frac{\partial^2 \Phi}{\partial z^k \partial \bar{z}^l} \] (1)

where \( z^i, \bar{z}^i \) (\( i = 1, 2, \ldots, N \)) are complex local coordinates.

In this article, we use the Einstein summation convention over repeated indices. The \( g^{kl} \) is the inverse of the Kähler metric tensor \( g_{kl} \). That means \( g^{kl} g_{lm} = \delta_{km} \).