CLIFFORD ALGEBRAS AND THEIR APPLICATIONS TO LIE GROUPS AND SPINORS

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Abstract. We discuss some well-known facts about Clifford algebras: matrix representations, Cartan’s periodicity of $8$, double coverings of orthogonal groups by spin groups, Dirac equation in different formalisms, spinors in $n$ dimensions, etc. We also present our point of view on some problems. Namely, we discuss the generalization of the Pauli theorem, the basic ideas of the method of averaging in Clifford algebras, the notion of quaternion type of Clifford algebra elements, the classification of Lie subalgebras of specific type in Clifford algebra, etc.

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CONTENTS

Introduction ................................................................. 12
1. Definition of Clifford Algebra .................................... 13
   1.1. Clifford Algebra as a Quotient Algebra ...................... 13
   1.2. Clifford Algebra with Fixed Basis ............................. 13
   1.3. Examples in Small Dimensions ............................... 15
2. Gradings and Conjugations ........................................ 16
   2.1. Gradings ...................................................... 16
   2.2. Center of Clifford Algebra ................................ 17
   2.3. Operations of Conjugations ................................ 18
   2.4. Quaternion Types of Clifford Algebra Elements .......... 19
3. Matrix Representations of Clifford Algebras ................. 21
   3.1. Cartan’s Periodicity of $8$, Central and Simple Algebras 21
   3.2. Clifford Trigonometry Circle and Exterior Signature of Clifford Algebra ... 24