



CLIFFORD ALGEBRAS AND THEIR APPLICATIONS TO LIE GROUPS AND SPINORS

DMITRY SHIROKOV

Higher School of Economics, National Research University, 101000 Moscow, Russia

Abstract. We discuss some well-known facts about Clifford algebras: matrix representations, Cartan’s periodicity of 8, double coverings of orthogonal groups by spin groups, Dirac equation in different formalisms, spinors in n dimensions, etc. We also present our point of view on some problems. Namely, we discuss the generalization of the Pauli theorem, the basic ideas of the method of averaging in Clifford algebras, the notion of quaternion type of Clifford algebra elements, the classification of Lie subalgebras of specific type in Clifford algebra, etc.

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CONTENTS

Introduction	12
1. Definition of Clifford Algebra	13
1.1. Clifford Algebra as a Quotient Algebra	13
1.2. Clifford Algebra with Fixed Basis	13
1.3. Examples in Small Dimensions	15
2. Gradings and Conjugations	16
2.1. Gradings	16
2.2. Center of Clifford Algebra	17
2.3. Operations of Conjugations	18
2.4. Quaternion Types of Clifford Algebra Elements	19
3. Matrix Representations of Clifford Algebras	21
3.1. Cartan’s Periodicity of 8. Central and Simple Algebras	21
3.2. Clifford Trigonometry Circle and Exterior Signature of Clifford Algebra ...	24