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STAR PRODUCT, STAR EXPONENTIAL AND APPLICATIONS

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Abstract. We introduce star products for certain function space containing polynomials, and then we obtain an associative algebra of functions. In this algebra we can consider exponential elements, which are called star exponentials. Using star exponentials we can define star functions in the star product algebra. We explain several examples.

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1. Introduction: The Idea

The idea of star product is deeply related to the canonical commutation relation in Quantum mechanics, which is given by a pair of operators $\hat{p}, \hat{q}$ such that

$$[\hat{p}, \hat{q}] = \hat{p}\,\hat{q} - \hat{q}\,\hat{p} = \sqrt{-1}\,\hbar = \mathrm{i}\,\hbar$$

where $\hat{p} = \mathrm{i}\,\hbar\,\partial_q$ and $\hat{q}$ is a multiplication operator $q\times$ acting on the functions of q, and $\hbar$ is a constant equal to the Planck constant divided by 2π. The algebra generated by $\hat{p}$ and $\hat{q}$ is called the *Weyl algebra* which plays a fundamental role in quantum mechanics.

We have another way to produce the same algebra without using operators. The idea is to introduce an associative product into the space of functions of (q, p). The product is different from the usual multiplication of functions, but is given by a deformation of the usual multiplication in the following way. (cf. Bayen-Flato-Fronsdal-Lichnerowicz-Sternheimer [1], Moyal [9]).

For smooth functions f, g on $\mathbb{R}^2$, we have the canonical Poisson bracket

$$\{f, g\}(q, p) = \partial_p f \partial_q g - \partial_q f \partial_p g, \qquad (q, p) \in \mathbb{R}^2.$$