Second International Conference on
Geometry, Integrability and Quantization
June 7–15, 2000, Varna, Bulgaria
Ivaïlo M. Mladenov and Gregory L. Naber, Editors
Coral Press, Sofia 2001, pp 171-180

SIGMA MODELS, MINIMAL SURFACES AND SOME RICCI FLAT PSEUDO-RIEMANNIAN GEOMETRIES

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Abstract. We consider the sigma models where the base metric is proportional to the metric of the configuration space. We show that the corresponding sigma model equation admits a Lax pair. We also show that this type of sigma models in two dimensions are intimately related to the minimal surfaces in a flat pseudo-Riemannian 3-space. We define two dimensional surfaces conformally related to the minimal surfaces in flat three dimensional geometries which enable us to give a construction of the metrics of some even dimensional Ricci flat (pseudo-) Riemannian geometries.

1. Introduction

Let M be a 2-dimensional manifold with local coordinates $x^\mu = (x, y)$ and $\Lambda^{\mu\nu}$ be the components of a tensor field in M. Let P be an $2{\times}2$ matrix with a nonvanishing constant determinant. We assume that P is a Hermitian ($P^\dagger = P$) matrix. Then the field equations of the sigma-model we consider is given as follows

$$\frac{\partial}{\partial x^\alpha}\left(\Lambda^{\alpha\beta} P^{-1}\,\frac{\partial P}{\partial x^\beta}\right) = 0\,. \tag{1.1}$$

The integrability of the above equation has been studied in [1] where the matrix function P and the tensor $\Lambda^{\alpha\beta}$ were considered independent. The sigma model equation given above is integrable provided Λ satisfies the conditions

$$\partial_\alpha\left(\frac{1}{\sigma}\,\Lambda^{\alpha\beta}\partial_\beta\sigma\right) = 0\,, \qquad \partial_\alpha\left(\frac{1}{\sigma}\,\Lambda^{\beta\alpha}\partial_\beta\phi\right) = 0\,, \tag{1.2}$$