SOME EXAMPLES RELATED TO THE DELIGNE–SIMPSON PROBLEM∗

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Abstract. We consider the variety of \((p+1)\)-tuples of matrices \(M_j\) from
given conjugacy classes \(C_j \subset GL(n, \mathbb{C})\) such that \(M_1 \cdots M_{p+1} = I\).
This variety is connected with the Deligne–Simpson problem: give nec-
essary and sufficient conditions on the choice of the conjugacy classes
\(C_j \subset GL(n, \mathbb{C})\) so that there exist irreducible \((p+1)\)-tuples of matrices
\(M_j \in C_j\) whose product equals \(I\). The matrices \(M_j\) are interpreted as
monodromy operators of regular linear systems on Riemann’s sphere.
We consider among others cases when the dimension of the variety is
higher than the expected one due to the presence of \((p+1)\)-tuples with
non-trivial centralizers.

1. Introduction

1.1. The Deligne–Simpson Problem

In the present paper we consider some examples related to the Deligne–
Simpson Problem (DSP) which is formulated like this:

Give necessary and sufficient conditions upon the choice of the \(p+1\) conjugacy
classes \(c_j \subset gl(n, \mathbb{C})\), resp. \(C_j \subset GL(n, \mathbb{C})\), so that there exist irreducible
\((p+1)\)-tuples of matrices \(A_j \in c_j\), \(A_1 + \cdots + A_{p+1} = 0\), resp. of matrices
\(M_j \in C_j\), \(M_1 \cdots M_{p+1} = I\).

By definition, the weak DSP is the DSP in which the requirement of irreducibil-
ity is replaced by the weaker requirement the centralizer of the \((p+1)\)-tuple
of matrices to be trivial.
The matrices \(A_j\), resp. \(M_j\), are interpreted as matrices-residua of Fuchsian
systems on Riemann’s sphere (i.e. linear systems of ordinary differential equa-

∗To the memory of my mother.