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DISCRETIZATION IN NONCOMMUTATIVE FIELD THEORY

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Abstract. A discretization scheme provided by the noncommutativity of space is reviewed. In the representation chosen here the radial coordinate is rendered discrete, allowing fields to be put on a lattice in a natural way. Noncommutativity is traded for a controllable type of nonlocality of the field dynamics, which in turn allows fermions to be free of lattice artefacts. Exact, singularity-free solutions are found interpreted, and their continuum limit is well-defined.

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1. Introduction

Exact results are difficult to obtain in nontrivial field theories. The only systematic procedure to date is numerical and relies on the Wilson lattice formulation of field theories [4]. In spite of four decades of extensive work, conceptual and technical issues persist in this approach, including the nature of the continuum limit and the doubling of fermionic degrees of freedom. We discuss here an alternative discretization scheme which puts to use field theories (FT) defined over noncommutative (NC) spaces. Suitable representations of the NC algebra induce a natural discrete structure, allowing to trade noncommutativity for a nonlocal discrete FT. Yet that peculiar nonlocality is controllable, as illustrated by finding and interpreting general solutions of the equations of motion. Following a preliminary discussion of scalar fields, discrete fermions are introduced naturally [1], without the (in)famous doubling problem. A complete discussion of the generic, nonlocal solutions [1, 2] is then given. Their nonlocality is precisely related to the angular momentum of the field configuration. The classical divergences of usual FT do not appear, unless of course one takes the commutative (continuum) limit, which