

CAYLEY–KLEIN POISSON HOMOGENEOUS SPACES

FRANCISCO J. HERRANZ[†], ANGEL BALLESTEROS[†], IVAN
GUTIERREZ–SAGREDO[†] and MARIANO SANTANDER[‡]

[†]*Departamento de Física, Universidad de Burgos, 09001 Burgos, Spain*

[‡]*Departamento de Física Teórica and IMUVa, Universidad de Valladolid
47011 Valladolid, Spain*

Abstract. The nine two-dimensional Cayley–Klein geometries are firstly reviewed by following a graded contraction approach. Each geometry is considered as a set of three symmetrical homogeneous spaces (of points and two kinds of lines), in such a manner that the graded contraction parameters determine their curvature and signature. Secondly, new Poisson homogeneous spaces are constructed by making use of certain Poisson–Lie structures on the corresponding motion groups. Therefore, the quantization of these spaces provides noncommutative analogues of the Cayley–Klein geometries. The kinematical interpretation for the semi-Riemannian and pseudo-Riemannian Cayley–Klein geometries is emphasized, since they are just Newtonian and Lorentzian spacetimes of constant curvature.

MSC: 17Bxx, 22Exx, 16Txx

Keywords: Contraction, curvature, deformation, Lorentzian spacetimes, Poisson–Lie groups, quantum groups, Riemannian geometries

CONTENTS

1. Introduction	162
2. The Nine Two-Dimensional Cayley–Klein Geometries	163
3. Kinematical Cayley–Klein Spaces	167
4. Generalized Dualities	168
5. Vector Model and Geodesic Coordinates for the Space of Points	171
6. Quantum Groups and Poisson Homogeneous Spaces	176
7. Cayley–Klein Poisson Homogeneous Spaces of Points	178
Acknowledgements	181
References	181