

QUASI-CLASSICAL CALCULATION OF EIGENVALUES BY MASLOV QUANTIZATION CONDITION

TOMOYO KANAZAWA and AKIRA YOSHIOKA

*Department of Mathematics, Tokyo University of Science, Kagurazaka 1-3
 Shinjuku-ku, Tokyo 162-8601, Japan*

Abstract. The Maslov quantization condition is a condition for Lagrangian submanifolds which is regarded as a mathematical extension of the Bohr-Sommerfeld quantization condition. In this survey note, we apply the Maslov quantization condition to several concrete Schrödinger operators and quantize invariant Lagrangian submanifolds of their classical systems. We see the quasi-classical energy levels are equal to the quantum ones for these operators and also the number of Lagrangian submanifolds is equal to the multiplicities of eigenvalues for these operators.

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