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MÖBIUS–LIE GEOMETRY AND ITS EXTENSION

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Abstract. This paper is a review of the classical Möbius–Lie geometry and recent works on its extension. The latter considers ensembles of cycles (quadrics), which are interconnected through conformal-invariant geometric relations (e.g. "to be orthogonal", "to be tangent", etc.), as new objects in an extended Möbius–Lie geometry. It is shown on examples, that such ensembles of cycles naturally parameterise many other conformally-invariant families of objects, two examples–the Poincaré extension and continued fractions are considered in detail. Further examples, e.g. loxodromes, wave fronts and integrable systems, are discussed elsewhere.

The extended Möbius–Lie geometry is efficient due to a method, which reduces a collection of conformally invariant geometric relations to a system of linear equations, which may be accompanied by one fixed quadratic relation. The algorithmic nature of the method allows to implement it as a C^{++} library, which operates with numeric and symbolic data of cycles in spaces of arbitrary dimensionality and metrics with any signatures. Numeric calculations can be done in exact or approximate arithmetic. In the two- and three-dimensional cases illustrations and animations can be produced. An interactive Python wrapper of the library is provided as well.

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