

ON QUASI-MINIMAL ISOMETRIC IMMERSIONS INTO NON-FLAT SEMI RIEMANNIAN SPACE FORMS OF INDEX TWO

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Abstract. In this paper, first we gave a summary of recent results on biconservative immersions. Then, we obtained a necessary and sufficient condition for the existence of biconservative quasi-minimal immersions into a four dimensional semi-Riemannian space form of index two with non-zero sectional curvatures. We also constructed an explicit example of biconservative quasi-minimal surface.

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1. Introduction

The energy functional E between two semi-Riemannian manifolds M, N is defined by

$$E(\phi) = \frac{1}{2} \int_M \|d\phi\|^2 dM$$

for a smooth map $\phi : M \hookrightarrow N$. The map ϕ is called a *harmonic* map if it is a critical point of the energy functional E . This condition is equivalent to satisfying the equation

$$\tau_1(\phi) := \text{trace } \nabla d\phi = 0$$