

SOLUTIONS TO A VECTOR HEISENBERG FERROMAGNET EQUATION RELATED TO SYMMETRIC SPACES

TIHOMIR VALCHEV and ALEXANDAR YANOVSKI[†]

*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
 Acad. G. Bonchev Str., Block 8, 1113 Sofia, Bulgaria*

[†]*Department of Mathematics & Applied Mathematics, University of Cape Town
 Rondebosch 7700, Cape Town, South Africa*

Abstract. In this report we consider a vector generalization of Heisenberg ferromagnet equation. That completely integrable system is related to a spectral problem in pole gauge for the Lie algebra $\mathfrak{sl}(n + 1, \mathbb{C})$. We construct special solutions over constant background using dressing technique.

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1. Introduction

In [8], the authors of the current text introduced the following matrix system of completely integrable equations

$$i\mathbf{u}_t + \left[(\mathbf{u}Q_m\mathbf{u}^\dagger Q_n)_x \mathbf{u} - \mathbf{u}(Q_m\mathbf{u}^\dagger Q_n\mathbf{u})_x \right]_x = 0 \quad (1)$$

and the corresponding auxiliary spectral problem. Above, the subscripts mean partial differentiation, “ \dagger ” denotes Hermitian conjugation and Q_m, Q_n are diagonal matrices of dimension m and n respectively having ± 1 on their principal diagonals. It is also assumed that the $n \times m$ matrix $\mathbf{u}(x, t)$ fulfill certain algebraic condition, see [8] for more details.

System (1) contains as particular cases the classical $1 + 1$ dimensional Heisenberg ferromagnet equation, known to be integrable through inverse spectral transform [2, 6] and some of its integrable generalizations recently studied [1, 9, 10]. Its Lax