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HIERARCHIES OF SYMPLECTIC STRUCTURES FOR $\mathfrak{sl}(3,\mathbb{C})$ ZAKHAROV-SHABAT SYSTEMS IN CANONICAL AND POLE GAUGE WITH $\mathbb{Z}_2\times\mathbb{Z}_2$ REDUCTION OF MIKHAILOV TYPE

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Abstract. We consider the theory of the hierarchies of nonlinear evolution equations associated with two gauge-equivalent auxiliary systems. They are obtained from the Generalized Zakharov-Shabat system on $\mathfrak{sl}(3,\mathbb{C})$ in general position making a $\mathbb{Z}_2\times\mathbb{Z}_2$ reductions of Mikhailov type in canonical and in pole gauge respectively. Using the Recursion Operators approach we study the symplectic structures of the hierarchies of the nonlinear evolution equations associated with the above auxiliary systems and find the relation between them.

MSC: 35Q51, 37K05, 37K10
Keywords: Lax representation, recursion operators

The concept of gauge-equivalent soliton equations originates from the paper [15] of Zakharov and Mikhailov in which they considered some field theory models. The next example was the gauge equivalence between the nonlinear Schrodinger equation (NLS) and the Heisenberg Ferromagnet (HF) equation, [16]. The NLS has Lax representation $[L_c, A_c] = 0$ where L_c is the linear differential operator (in ∂_x) that enters in the classical Zakharov-Shabat 2×2 matrix spectral problem $L_c\psi = 0$ and A_c is 2×2 linear differential operator (in ∂_t) with quadratic dependence on the spectral parameter λ. From its side HF has Lax representation $[\tilde{L}_c, \tilde{A}_c] = 0$ with operators $\tilde{L}_c, \tilde{A}_c$ that have similar dependence on ∂_t, ∂_x and λ. In [16] Zakharov and Takhtadjan showed that HF and NLS are gauge-equivalent, that is $\tilde{L}_c = \Psi_0^{-1}L_c\Psi_0$ and $\tilde{A}_c = \Psi_0^{-1}A_c\Psi_0$, where Ψ_0 is the Jost solution of $L_c\Psi_0 = 0$ for $\lambda = 0$ having some additional proprieties that ensure the necessary asymptotic behavior of the coefficients of $\tilde{L}_c$. Thus in the theory of the soliton equations the gauge-equivalence started to be considered as a sort of changing