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Geometry, Integrability and Quantization

WILLMORE-LIKE ENERGIES AND ELASTIC CURVES WITH POTENTIAL

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Abstract. We study invariant Willmore-like tori in total spaces of Killing submersions. In particular, using a relation with elastic curves with potentials in the base surfaces, we analyze Willmore tori in total spaces of Killing submersions. Finally, we apply our findings to construct foliations of these total spaces by constant mean curvature Willmore tori.

MSC: 53B20, 53A10
Keywords: Elasticae with potential, Willmore-like energies, Willmore tori

1. Introduction

The purpose of this paper is to present some results included in [3] where a complete and more general treatment can be found. Here, the ideas and arguments are only sketched while proofs are omitted.

One of the most important tools to understand the extrinsic geometry of an immersed surface into an ambient space is, probably, the *mean curvature*. This term was first coined by S. Germain who considered the integral of the squared mean curvature as a proposal to measure the free energy controlling the physical system associated with an elastic plate, [8].

This energy is usually called *total squared curvature* or *bending energy*. In particular, for a surface S immersed into the Euclidean three-space $\mathbb{R}^3$, the bending energy is given by

$$\mathcal{W}(S) = \int_S H^2 \, \mathrm{d}A$$

where the mean curvature of S is denoted by H. The variational problem associated to this bending energy was studied in the decade of 1920 by Blaschke's