

ON THE NONCOMMUTATIVE FEYNMAN PROBLEM

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Abstract. We extend the Feynman derivation of the Maxwell-Lorentz equations to the case in which coordinates do not commute, adding significantly to previous results. New dynamics is pinned down precisely both at the level of the homogeneous equations and for the Lorentz force, for which a complete derivation is given for the first time.

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1. Introduction and Summary

R. P. Feynman's alternative view of electromagnetic fields, as reported by F. J. Dyson [10], relies on a few quite general assumptions, cf. equations (1)–(3) below. Somewhat unexpectedly, these simple hypotheses enforce precisely the homogeneous Maxwell equations and the Lorentz force law. This success however precludes Feynman's initial goal, of providing a minimalist framework capable to point at new physics. Extensions involving additional scalar or tensor indices appear to confirm this, as they indeed mainly describe Yang-Mills fields [11, 15–17] and gravitational fields [16]. Further developments [9], although of interest, do not appear to suggest fundamentally new dynamical possibilities either.

Although Feynman deliberately avoided the standard variational formulation [10] his hypotheses turned out to actually guarantee it [12, 13]. The commutativity assumption, equation (1) below, is in fact essential [12] for the existence of a