CAYLEY MAP FOR SYMPLECTIC GROUPS

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Abstract. Despite of their importance, the symplectic groups are not so popular like orthogonal ones as they deserve. The only explanation of this fact seems to be that their algebras can not be described so simply. While in the case of the orthogonal groups they are just the anti-symmetric matrices, those of the symplectic ones should be split in four blocks that have to be specified separately. It turns out however that in some sense they can be presented by the even dimensional symmetric matrices. Here, we present such a scheme and illustrate it in the lowest possible dimension via the Cayley map.

Besides, it is proved that by means of the exponential map all such matrices generate genuine symplectic matrices.

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1. Introduction

Lie groups can be considered as the best mirrors that reflect the geometry of the real word. Among them the symplectic groups and their Lie algebras are of immense importance for the modern physics. It is enough just to mention their appearances in classical and quantum mechanics [4, 5, 10, 15], nuclear physics [7, 13], optics [9, 12, 20], etc.

Another class of no less importance is that one of the orthogonal groups [2, 17]. Both classes have been a subject of deep studies and have found countless applications. There exists however a serious distinction. While the Lie algebras of the