Twenty Second International Conference on
Geometry, Integrability and Quantization
June 8–13, 2020, Varna, Bulgaria
Ivaïlo M. Mladenov, Vladimir Pulov
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2021, pp 209–218
doi: 10.7546/giq-22-2021-209-218

STAR PRODUCT ON THE EUCLIDEAN MOTION GROUP IN THE PLANE

LAARNI B. NATIVIDAD and JOB A. NABLE†

Mathematics Department, Saint Louis University, 2600 Baguio City, Philippines

†*Mathematics Department, Ateneo De Manila University, 1105 Quezon City, Philippines*

Abstract. In this work, we perform exact and concrete computations of star-product of functions on the Euclidean motion group in the plane, and list its C-star-algebra properties. The star-product of phase space functions is one of the main ingredients in phase space quantum mechanics, which includes Weyl quantization and the Wigner transform, and their generalizations. These methods have also found extensive use in signal and image analysis. Thus, the computations we provide here should prove very useful for phase space models where the Euclidean motion groups play the crucial role, for instance, in quantum optics.

MSC: 81S30, 20E35, 22E70
Keywords: Euclidean motion group, Moyal star-product, unitary irreducible representation, Weyl transform, Wigner function

1. Introduction

Quantization, as we see it, is best viewed as the study of possible correspondences of a given classical system and a given quantum system, and not as a search for a particular linear mapping $\mathcal{Q}$ from the space of classical observables to the set of quantum observables. On one side is Hamiltonian mechanics and on the other is deformation quantization. The correspondence relies crucially on dynamical symmetry groups acting on certain subsets of Lie algebra duals, called coadjoint orbits, insofar as generalizations to nonflat phase spaces are concerned, including the discrete and finite cases. Multiplication and bracket in deformation algebras are deformation of the ordinary commutative product and the Poisson bracket of