Twenty Second International Conference on
Geometry, Integrability and Quantization
June 8–13, 2020, Varna, Bulgaria
Ivaïlo M. Mladenov, Vladimir Pulov
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2021, pp 274–285
doi: 10.7546/giq-22-2021-274-285

MULTICOMPONENT NONLINEAR EVOLUTION EQUATIONS OF THE HEISENBERG FERROMAGNET TYPE: LOCAL VERSUS NONLOCAL REDUCTIONS

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Abstract. This work is dedicated to systems of matrix nonlinear evolution equations related to Hermitian symmetric spaces of the type $\mathbf{A.III}$. The systems under consideration generalize the $1 + 1$ dimensional Heisenberg ferromagnet equation in the sense that their Lax pairs are linear bundles in pole gauge like for the original Heisenberg model. Here we present certain local and nonlocal reductions. A local integrable deformation and some of its reductions are discussed as well.

MSC: 35G50, 37K10
Keywords: generalized Heisenberg ferromagnet equation, integrable deformations, nonlocal reductions

1. Introduction

Let us consider the following matrix system

$$\begin{aligned} \mathrm{i}\mathbf{u}_t + \left(\mathbf{u}_x\mathbf{v}^T\mathbf{u} - \mathbf{u}\mathbf{v}^T\mathbf{u}_x\right)_x = 0 \\ \mathrm{i}\mathbf{v}_t - \left(\mathbf{v}_x\mathbf{u}^T\mathbf{v} - \mathbf{v}\mathbf{u}^T\mathbf{v}_x\right)_x = 0 \end{aligned} \tag{1}$$

where the subscripts in x and t stand for partial derivatives in spatial and time variable respectively, the superscript T denotes matrix transposition and "i" is the imaginary unit. The rectangular complex matrices $\mathbf{u}(x,t)$ and $\mathbf{v}(x,t)$ are not independent but are required to fulfil the constraints

$$\mathbf{u}\mathbf{v}^T\mathbf{u} = \mathbf{u}, \qquad \mathbf{v}\mathbf{u}^T\mathbf{v} = \mathbf{v}.$$