

REAL FORMS OF COMPLEXIFIED HAMILTONIAN DYNAMICS

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Abstract. Complexified Hamiltonian dynamical systems are considered with subsequent construction of real forms of Hamiltonian dynamics by using compatible complex structures or involution operators mimicking the properties of complex conjugation. This provides a method of associating a class of real dynamical systems to a given initial (also real) one. Examples are given and the problem of integrability of the derived dynamical systems is also discussed.

1. Introduction

We start with a real Hamiltonian system (\mathcal{M}, ω, H) with n degrees of freedom and Hamiltonian H depending analytically on the dynamical variables. Such systems can be complexified and then considered as Hamiltonian systems with $2n$ (real) degrees of freedom. Our main construction relates to each compatible involutive automorphism \tilde{C} of the complexified phase space and commuting with the complex structure a real Hamiltonian form of the complexified system. Just like with each complex Lie algebra one associates several inequivalent real forms, so to each complexified dynamics we associate several inequivalent real forms which again have n real degrees of freedom just like the initial system. Provided $\tilde{C}(H) = H$ the dynamics on the real form will be well defined and