Fourth International Conference on
Geometry, Integrability and Quantization
June 6–15, 2002, Varna, Bulgaria
Ivaïlo M. Mladenov and Gregory L. Naber, Editors
Coral Press, Sofia 2003, pp 326–329

ON LOCALLY LAGRANGIAN SYMPLECTIC STRUCTURES

IZU VAISMAN

Department of Mathematics, University of Haifa
31905 Haifa, Israel

Abstract. Some results on global symplectic forms defined by local Lagrangians of a tangent manifold, studied earlier by the author, are summarized without proofs.

This is a summary of some of our results on locally Lagrange symplectic and Poisson manifolds [3, 4].

The symplectic forms used in Lagrangian dynamics are defined on tangent bundles TN, and they are of the type

$$\omega_{\mathcal{L}} = \sum_{i,j=1}^{n} \left(\frac{\partial^2 \mathcal{L}}{\partial x^i \partial \xi^j} \, \mathrm{d}x^i \wedge \mathrm{d}x^j + \frac{\partial^2 \mathcal{L}}{\partial \xi^i \partial \xi^j} \, \mathrm{d}\xi^i \wedge \mathrm{d}x^j \right) \tag{1}$$

where $(x^i)_{i=1}^n$ $(n = \dim N)$ are local coordinates on N, (ξ^i) are the corresponding natural coordinates on the fibers of TN, and $\mathcal{L} \in C^\infty(TN)$ is a non degenerate Lagrangian.

An **almost tangent structure** on a differentiable manifold M^{2n} is a tensor field $S \in \Gamma\,\mathrm{End}(TM)$ (necessarily of rank n) such that

$$S^2 = 0, \qquad \mathrm{Im}\, S = \mathrm{Ker}\, S\,. \tag{2}$$

If the Nijenhuis tensor vanishes, i. e. $\forall X, Y \in \Gamma TM$,

$$\mathcal{N}_S(X,Y) = [SX, SY] - S[SX, Y] - S[X, SY] + S^2[X,Y] = 0, \tag{3}$$

S is a **tangent structure**. Then, $V = \mathrm{Im}\, S$, is an integrable subbundle, and we call its tangent foliation the **vertical foliation** $\mathcal{V}$. Furthermore, M has local