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FERMIONS AND SUPERSYMMETRY

ALLEN C. HIRSHFELD

Fachbereich Physik, Universität Dortmund
44221 Dortmund

Abstract. Fermions and bosons are the fundamental particles of nature, and they are naturally described mathematically by using the methods of supersymmetry. These methods are illustrated here by consideration of a number of physical examples which arise in non-relativistic and relativistic quantum mechanics.

1. Introduction

Matter occurs in nature in two forms: particles of integral spin are *bosons*, particles of half-integral spin are *fermions*. Bosons and fermions obey different statistics: Bosons obey Bose-Einstein statistics, fermions obey Fermi-Dirac statistics. In quantum field theory this difference arises because the bosons obey fundamental equal-time *commutator relations*, whereas fermions obey *anticommutation relations*. At the macroscopic level these differences lead to spectacular effects: Fermi-Einstein statistics and the associated Pauli exclusion principle is responsible for the existence of the shell structure of atoms and nuclei. On the other hand, Bose-Einstein statistics lead to the phenomena of *Bose-Einstein condensation*, which was directly observed for the first time in 2001. The discoverers were awarded the Nobel Prize in Physics in 2001 [4]. The Nobel Prize in 2003 honoured the theorists who succeeded in explaining the phenomena of superconductivity and superfluidity, which are also due to the Bose-Einstein condensation of Cooper pairs [1].

In order to achieve a unified treatment of bosons and fermions in theoretical physics new methods which have their origins in supersymmetric field theories are necessary. While the supersymmetry predicted in field theory for elementary particles has not yet been observed, it has been observed in nuclear physics [11]. It has also become an important tool in quantum mechanics [3], in atomic, condensed matter and statistical physics [12], in the description of gauge theories [2], and in mathematics [10].