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ON PHASE SPACES AND THE VARIATIONAL BICOMPLEX (AFTER G. ZUCKERMAN)

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Abstract. The notion of a phase space in classical mechanics is of course well known. The extension of this concept to field theory however, is a challenging endeavor, and over the years numerous propositions for such a generalization have appeared in the literature. In this contribution we review a Hamiltonian formulation of Lagrangian field theory based on an extension to infinite dimensions of J.-M. Souriau's symplectic approach to mechanics. Following G. Zuckerman, we state our results in terms of the variational bicomplex. We present a basic example, and briefly discuss some possible avenues of research.

1. Introduction

It appears it was H. Bacry [3] who first noted that one can find the equations of motion of (spinning) elementary particles by studying Hamiltonian systems on coadjoint orbits of the Poincaré group. By doing so, he realized that it is natural and important to introduce phase spaces not just as a set of p's and q's equipped with the canonical form $\mathrm{d}q^i \wedge \mathrm{d}p_i$, but as non-trivial symplectic manifolds. His work was put in a general context by J.-M. Souriau in his ground-breaking *Structure des Systemes Dynamiques* [18]. This treatise is the first complete treatment of mechanics which fully utilizes the language and techniques of symplectic geometry.

It is now widely recognized that a fructiferous approach for treating dynamical problems with a finite number of degrees of freedom, is to model them as Hamiltonian systems on (in general non-trivial) symplectic manifolds [2, 7, 12, 13]. It is less clear how to proceed when considering field theory. A completely rigorous point of view based on manifolds modelled on Banach (or Frechet) spaces would perhaps be the approach of choice, but to pursue such an endeavor is very delicate: