

SINGLE-VALUED AND MULTIVALUED SOLUTIONS FOR THE GENERALIZED HÉNON–HEILES SYSTEM WITH AN ADDITIONAL NONPOLYNOMIAL TERM

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Abstract. The generalized Hénon–Heiles system with an additional non-polynomial term is considered. The standard method for the search of the elliptic solutions is a transformation of an initial nonlinear polynomial differential equation into a nonlinear algebraic system. It has been demonstrated that the use of the Laurent-series solutions allows to simplify the resulting algebraic system. This procedure has been automatized and generalized on some type of multivalued solutions. To find solutions of the initial equation in the form of the Laurent or Puiseux series we use algorithm of the Painlevé test.

1. The Painlevé Property

Let us formulate the Painlevé property for ordinary differential equations (ODE's). Solutions of a system of ODE's are regarded as analytic functions, maybe with isolated singular points [9]. A singular point of a solution is said **critical** (as opposed to **noncritical**) if the solution is multivalued (single-valued) in its neighborhood and *movable* if its location depends on initial conditions. The **general solution** of an ODE of order N is the set of all solutions mentioned in the existence theorem of Cauchy, i.e. determined by the initial values. It depends on N arbitrary independent constants. A **special solution** is any solution obtained from the general solution by giving values to the arbitrary constants. A **singular solution** is any solution which is not special, i.e. which does not belong to the general solution. A system of ODE's has **the Painlevé property** if its general solution has no movable critical singular point [9, 14].

There exist two distinctions between the structure of solutions of linear differential equations and nonlinear ones. Linear ODE's have no singular solution and their general solutions have no movable singularity.